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Chapter 10

The Binomial Theorem

Lesson 10-3 (pp. 631-636)

Mental Math

a. 144

b. $x^2 + 2xy + y^2$

c. 1681

Activity

Step 1:

```
expand((x+y)^1)                          x+y
expand((x+y)^2)                  x^2+2·x·y+y^2
expand((x+y)^3)      x^3+3·x^2·y+3·x·y^2+y^3
expand((x+y)^4)
          x^4+4·x^3·y+6·x^2·y^2+4·x·y^3+y^4
expand((x+y)^5)
```

$(x + y)^5 = x^5 + 5x^4y + 10x^3 + 10x^2y^3 + 5xy^4 + y^5$

Step 2: The coefficients are the numbers in the nth row of Pascal's triangle.

1; 1; 3; 3; 1; 1; 4; 6; 4; 1; 1; 5; 10; 10; 5; 1; 1; 6; 15; 20; 15; 6; 1

Step 3: $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

```
expand((x+y)^5)
 x^5+5·x^4·y+10·x^3·y^2+10·x^2·y^3+5·x·y^4+▸
expand((x+y)^6)
 x^6+6·x^5·y+15·x^4·y^2+20·x^3·y^3+15·x^2·y▸
```

Step 4: Since any real number raised to the power 0 is 1, $(x + y)^0 = 1$; we can consider this equivalent to row 0 of Pascal's Triangle.

Guided Example 1

5; 5; 3; 56; 56

Questions

1. a. 4
   b. We can use the combination ${}_4C_1 = 4$ to find the coefficient because the coefficients of the terms in the binomial expansion are the numbers in the nth row of Pascal's Triangle.
2. -945
3. 64
4. a. ${}_{1008}C_{11}$
   b. 25,889,925,543,680,106,113,914,800
5. $1728p^3q$
6. $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$

   ```
   expand((x-2)^6)
    x^6-12·x^5+60·x^4-160·x^3+240·x^2-192·▸
   ```

   $0^6 = 0$; $64 - 384 + 960 - 1280 + 960 - 384 + 64 = 0$
7. ```
   expand((1-3·x)^4)
          81·x^4-108·x^3+54·x^2-12·x+1
   ```

   $(1 - 3 \cdot 2)^4 = (-5)^4 = 625$; $1296 - 864 + 216 - 24 + 1 = 625$
8. ```
   expand((2·x+1)^5)
       32·x^5+80·x^4+80·x^3+40·x^2+10·x+1
   ```

   $(2 \cdot 2 + 1)^5 = (5)^5 = 3125$; $1024 + 1280 + 640 + 160 + 20 + 1 = 3125$
9. a. 20
   b. 42
10. a. true
    b. false
11. $(10^3)^4 + 4(10^3)^3 + 6(10^3)^2 + 4(10^3) + 1 =$ 1,004,006,004,001
12. $1 - 4(0.1) + 6(0.1)^2 - 4(0.1)^3 + (0.1)^4 = 0.6561$
13. a. 15
    b. $\frac{1}{64}$
    c. $\frac{81}{4096}$
14. a. 6
    b. $\frac{3}{8}$
15. Let $2^n = (1 + 1)^n$. By the Binomial Theorem, for any nonnegative integer n, $(1 + 1)^n = {}_nC_0 + \cdots + {}_nC_{n-1} + {}_nC_n = \sum_{k=0}^{n} {}_nC_k$.
16. a. 210
    b. This involves counting the number of ways 3 x's, 2 y's, and 2 z's can be arranged into a 7 letter string.
17. 462
18. a. 462, 330, 165, 55, 11, 1
    b. 1, 12, 66, 220, 495, 792, 924, 792, 495, 220, 66, 12, 1
    c. 1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1

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19. a. permutation
    b. 994,010,994,000
20. a. combination
    b. 41,417,124,750
21. $\frac{34}{_{36}C_3} = \frac{1}{210}$
22. a. $1 + 5(0.001) + 10(0.001)^2 + 10(0.001)^3 + 5(0.001)^4 + (0.001)^5 = 1.005010$
    b. 2
    c. $1 + 8(0.002) + 28(0.002)^2 = 1.016112$
    d. 1.01611244912
    e. The estimate is accurate to the nearest millionth.